PU-003-1164004 Seat No. ________

M. Sc. (Sem. IV) (CBCS) Examination

August - 2020

CMT - 4004 : Mathematics

(Graph Theory)

Faculty Code : 003
Subject Code : 1164004

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) Each question carries 14 marks.

1 Answer any **seven** questions : **7×2=14**

(i) Define terms : Degree of a vertex, pendent vertex in graph G and null graph.

(ii) Define subgraph of a graph G and draw a graph G with its two subgraphs H_1, H_2 so that H_1 and H_2 have five common vertices, but they have no common edges. $\left(i.e. \left|V(H_1) \cap V(H_2)\right| = and\ E(H_1) \cap E(H_2) = \phi\right)$.

(iii) Define k-regular graph. Also define terms $\delta(G)$ and $\Delta(G)$.

(iv) Define isomorphism of two graphs. Write down at least two properties for two isomorphic graphs G_1 and G_2.

(v) Define Hamiltonian cycle and draw wheel graph W_7 with its Hamiltonian cycle.

(vi) Define Eulerian graph. Draw a graph G, which admits an Eulerian line and draw another graph H, which can't admit any Eulerain line.

(vii) Define incidence matrix and write down the incidence matrix for the cycle C_4.

(viii) Write down at least three properties of adjacency matrix $X(G)$ for a graph G.

2 Answer any **two** questions : **2×7=14**

(a) Let G be a graph and it contains exactly two odd vertices, say $x, y \in V(G)$. Prove that x and y both lies in the same component of G.

(b) Let G be a simple graph with n vertices, q edges and k number of components in G. In standard notation prove that $q \leq \frac{1}{2}(n-k)(n-k+1)$.

(c) Let G be a connected graph with $E(G) \neq \phi$. Prove that G is an Eulerian graph if and only if it can be decomposed into edge disjoint cycles.

(d) State and prove Euler's Theorem.

3 Answer any **one** question : **1×14=14**

(a) For a simple connected planner graph G, derive Euler's formula $f = e - n + 2$ and also prove that (i) $e \geq \frac{3f}{2}$ (ii) $e \leq 3n - 6$. Using these prove that K_5 and $K_{3,3}$ both are non–planner graphs, where e = number of edges in G, n–number of vertices in G and f = number of faces in the planner graph G.

(b) Let G be a simple graph, $|V(G)| > 2 \text{ and } d_G(v) \geq \frac{n}{2}, \forall v \in V(G)$. Prove that G is a Hamiltonian graph. Also define closure of a graph G and write down closure for K_4, C_5.

4 Answer any **two** questions : **2×7=14**

(a) Define minimally connected graph. Prove that a graph G is a minimally connected graph if and only if it is a tree.

(b) Let T be a tree with $V(T) \neq \phi$. Prove that T has either one center two centers. In the case it has two centers they must be adjacent in T.

(c) Prove that a connected graph G, admits a spanning tree.

5 Answer any **two** questions : **2×7=14**

(a) Define weighted graph and minimal spanning tree. Write down two algorithms to obtain minimal spanning tree for a weighted connected graph G, in detail.

(b) Let G be a connected graph with $|V(G) > 2.|$ Prove that the vertex connectivity for $G \leq$ the edge connectivity for G.

(c) Define a separable graph. Prove that for a separable graph G, v is a cut vertex in G if and only if there are two vertices $x, y \in V(G) - \{v\}$ such that every path in G between x and y passes through v.

(d) Let T be a tree with $|V(T)| \geq 2$. Prove that T is a 2-chromatic graph.